MS221 : Supplementary Resource Material for Chapter C1

Question 1

Differentiate each of the following functions:

(a) $f(x) = x^2 e^x \tan x$ (b) $g(x) = \dfrac{x^2 \cot x}{e^x \sin x}$ (c) $h(x) = \dfrac{x^2 \ln x}{(x^2-1)^2}$

Question 2

Use the Composite Rule to differentiate each of the following functions:

(a) $f(x) = \sqrt{\sin^3 x}$ (b) $g(x) = \ln\left(e^{\sin x}\right)$ (c) $h(x) = \sec\left(\operatorname{cosec}^2 x\right)$

Question 3

Use the Inverse Rule to find each of the following derivatives:

(a) $\dfrac{d}{dx}\left(\operatorname{arcsec} x\right)$ (b) $\dfrac{d}{dx}\left(\operatorname{arccot} x\right)$ (c) $\dfrac{d}{dx}\left(\operatorname{arccosec} x\right)$

Question 4

Use the Composite Rule in conjunction with a table of standard derivatives to find each of the following:

(a) $\dfrac{d}{dx}\left(\arcsin\sqrt{x}\right)$ (b) $\dfrac{d}{dx}\left(\arctan\left(e^{2x}\right)\right)$ (c) $\dfrac{d}{dx}\left(\arccos\left(x^2\right)\right)$

Question 5

If $y = x^x$, prove that $\dfrac{dy}{dx} = x^x\left(1+\ln x\right)$ and that $\dfrac{d^2y}{dx^2} = x^{x-1} + x^x\left(1+\ln x\right)^2$

Question 6

Where appropriate, use the graph sketching strategy introduced on page 36 of Chapter *C*1 to sketch each of the following graphs:

(a) $y = \dfrac{x^2-x+2}{x^2-x+1}$ (b) $y = \left|\dfrac{x^2-x+2}{x^2-x+1}\right|$ (c) $y = \left|\dfrac{x-2}{2x+1}\right|$

(d) $y = x(x-1)(x-3)$ (e) $y^2 = x(x-1)(x-3)$ (f) $y = |x|\left(|x|-1\right)\left(|x|-3\right)$

Question 7

(a) By sketching the graphs of $y = e^{-x}$ and $y = x^2$ on the same set of axes, show that the equation $x^2 e^x = 1$ has exactly one real root.

(b) By applying the Newton-Rhapson method to the equation $f(x)=0$, where $f(x)=x^2e^x-1$, show that $x_{n+1}=x_n-\dfrac{x_n^2e^{x_n}-1}{x_ne^{x_n}\left(x_n+2\right)}$

(c) Use the Newton-Rhapson method with an appropriate starting value to find the real root of the equation $x^2e^x=1$, giving your answer to as many decimal places as your calculator allows.

(d) Explain how you can make your calculator automatically generate the sequence of iterates by repeatedly pressing the **ENTER** or **=** key.

Question 8

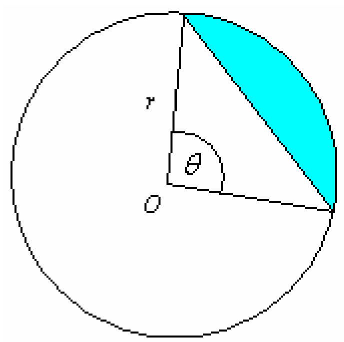
Use the Newton-Rhapson method to find all 4 roots of the quartic equation $2x^4-3x^3-68x^2+147x+89=0$, giving your answers accurate to as many decimal places as your calculator will allow.

Question 9

Consider a segment of a circle of radius r cm subtending an angle of θ radians at the centre. If the area of this segment is to be exactly $\frac{1}{20}$ of the area of the circle, show that

$$\theta-\sin\theta=\tfrac{1}{10}\pi$$

Hence, using the Newton-Rhapson method, give the value of θ correct to 8 places of decimals.



Question 10

(a) On the same set of axes, sketch the graphs of $y=\tan x$ and $y=3e^{2x}+10$ for $0\le x<\frac{3\pi}{2}$. Hence show that there is exactly one real root of the equation $\tan x=3e^{2x}+10$ on $\left[0,\frac{\pi}{2}\right)$.

(b) By applying the Newton-Rhapson method to the equation $f(x)=0$, where $f(x)=\tan x-3e^{2x}-10$, show that $x_{n+1}=x_n-\dfrac{\tan x_n-3e^{2x_n}-10}{\sec^2 x_n-6e^{2x_n}}$

(c) Taking $x_0=1.5$, compute the first 10 iterates $x_1,\ x_2\ldots x_{10}$ and comment on your results.

(d) Given that $x=1.5579199137808\ldots$, and that $f\,'(1.5579199137808\ldots)\neq 0$, explain why the sequence of iterates that you obtained in (c) failed to converge to the root of $f(x)=0$ on $\left[0,\frac{\pi}{2}\right)$.

(f) Show that the sequence of iterates does converge to the required root if we take $x_0=1.55$.