MST 121: Supplementary Resource Material for Chapter A2, Lines and Circles

1. Find the equation of the straight line that has gradient 4 and passes through the point $(-3,5)$.

2. Find the equation of the straight line that passes through the points $(-2,7)$ and $(1,1)$ (i) By using $y = mx + c$ (ii) By using $y - y_1 = m(x - x_1)$. In each case express your answer in the form $ax + by + c = 0$.

3. Find the equation of the straight line that passes through the points $(-5,-9)$ and $(2,-11)$ by formulating and solving a pair of linear simultaneous equations. Express your answer in the form $ax + by + c = 0$.

4. (a) Find the equation of the straight line that has x-intercept 6 and y-intercept 5.
   (b) Find the equation of the straight line that is parallel to $2x - 3y = -5$ and passes through the point $(-1,11)$. Express your answer in the form $ax + by + c = 0$.

5. What is the equation of the perpendicular bisector of the line joining (2,3) to (8,9)?

6. Find the point of intersection of the lines with equations $y = 3x - 4$ and $2x + 5y = 7$

7. Find the distance between the points $(2,10)$ and $(4,18)$, expressing your answer in exact surdform.

8. (a) Write down the equation of the circle with centre $(-1,2)$ and radius 5 in the form $(x-a)^2 + (y-b)^2 = r^2$.
   (b) Is the point (2,7) inside, outside or on the boundary of this circle?

9. Write down the centre and radius of the circle with equation $(x+2)^2 + (y-3)^2 = 8$. Is the point (1,5) inside, outside or on the boundary of this circle?

10. Let A, B and C be the points (1,5), (3,7) and $(8,-2)$ respectively.
    - Find the equations of the perpendicular bisectors of AB and BC
    - Solve these two equations simultaneously to find the co-ordinates of the points where these two lines meet.
    - Find the distance between this point of intersection and the point A, expressing your answer in exact surdform.
    - Hence write down the equation of the circle that passes through the points A, B and C.

11. An alternative method of solution is to substitute each of the co-ordinates in turn into the standard equation of the circle, $(x-a)^2 + (y-b)^2 = r^2$, and then solve the resulting three simultaneous equations for a, b, and r. Use this method to determine the equation of the circle in question 10, and verify that both methods of solution produce the same circle.

12. Find the centre and the radius of each of the following circles, and in each case state whether or not the point (1,3) is inside, outside or on the boundary of this circle?

    (i) $x^2 + y^2 - 6x + 4y - 3 = 0$ (ii) $6x^2 + 6y^2 + 36x - 42y - 72 = 0$

13. Find the co-ordinates of the points of intersection of each of the following lines and circles:

(a) $y + 2x = -5$ and $x^2 + y^2 - 6x + 10y - 4 = 0$

(b) $3x - 4y + 1 = 0$ and $2x^2 + 2y^2 + 8x - 4y + 1 = 0$

(c) $y = 7x - 11$ and $x^2 + y^2 + 4x - 9y = 4$

14. Use the method of question 13 to find the co-ordinates of the points of intersection of the line $2x - y = -1$ and the ellipse $x^2 + 2y^2 - 6x + 4y = 11$

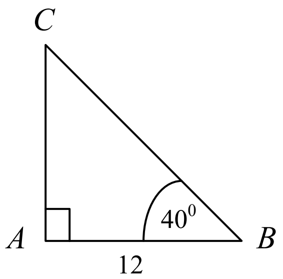
15. What is the equation of the line that is inclined at 45^0 to the direction of the positive x-axis and which passes through the point $(2,-5)$.

16. Complete the following table, expressing all irrational answers in exact surdform

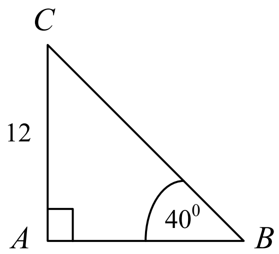
		Ratio					
Angle (Degrees)	**Angle (Radians)**	***sin***	***cos***	***tan***	***cosec***	***sec***	***cot***
0							
	$\frac{\pi}{6}$						
		$\frac{1}{\sqrt{2}}$					
				$\sqrt{3}$			
	$\frac{\pi}{2}$						
			-1				

17. Solve each of the following triangles

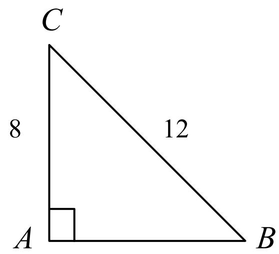
(a)



(b)



(c)



18. Write down parametric equations for

(a) The line which has gradient 4 and passes through the point $(2,-5)$

(b) The line which passes through the points $(2,6)$ and $(-3,-2)$.

19. (i) Eliminate t from the parametric equations $x = 7t + 2,\ 3y - 2x = 4t$.
(ii) Hence determine the angle (to the nearest degree) which the line defined by these parametric equations makes with the direction of the positive x-axis.

20. Solve the following sets of parametric equations

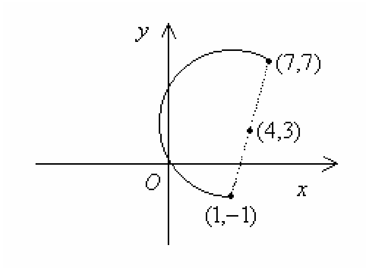
(a) $3x^2 = 4t - 1,\ 2y - 5 = x + 3t$ (b) $2x - 5t = -1,\ y + t = 0$

21. Two submarines travel in straight lines at the same depths and constant velocities. With reference to a fixed origin, the positions of the submarines are given by the co-ordinates $(5t - 4, 2t + 7)$ and $(4t + 12, t - 3)$, where distance is measured in kilometres and time in hours. What is the minimum distance between the two craft, and at what time does this position of closest approach occur?

22. Find a parametrisation of the unit circle which corresponds to motion anticlockwise at the rate of 3 revolutions per unit time.

23. Explain why $\cos(-\theta) = \cos(\theta)$, but $\sin(-\theta) = -\sin(\theta)$. What can we say about $\tan(-\theta)$?

24. (a) Write down parametric equations, including an appropriate range for the parameter t, for the given semi-circle.



(b) Verify that the Cartesian equation of the semi-circle is $x^2 - 8x + y^2 - 6y = 0$, for appropriate values of x and y, and that the equation of the diameter shown is $y = \frac{1}{3}(4x - 7)$.

(c) Solve these two simultaneous equations algebraically, and hence confirm that the points of intersection of the line and the circle are indeed $(1, -1)$ and $(7, 7)$.

25. In the previous question we saw that the equation of the semi-circle could be expressed in the form $x^2 - 8x + y^2 - 6y = 0$, and also parametrically. Use the trigonometric identity $\sin^2 t + \cos^2 t \equiv 1$ to show how the Cartesian equation could have been derived from the parametric form.

26. By firstly eliminating the parameter t, show that the parabola $y = 2 + 2x - x^2$ given in fig.(i) satisfies the parametric equations $x = 1 + t, y = 3 - t^2$. Write on your sketch the exact co-ordinates of any points where the curve intersects the co-ordinate axes, the co-ordinates of the vertex and the equation of the axis of symmetry?

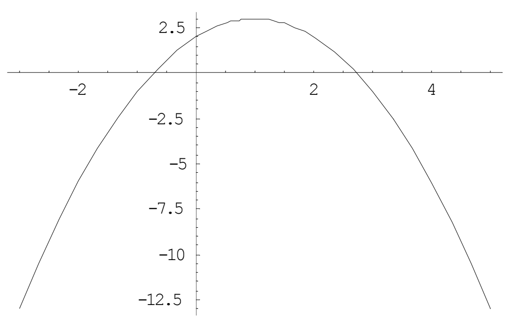


fig.(i)

Answers:

1. $y = 4x + 17$

2. $2x + y - 3 = 0$

3. $2x + 7y + 73 = 0$

4. (a) $5x + 6y - 30 = 0$ (b) $2x - 3y + 35 = 0$

5. $y = -x + 11$

6. $(x, y) = (\frac{27}{17}, \frac{13}{17})$

7. $2\sqrt{17}$

8. (a) $(x+1)^2 + (y-2)^2 = 25$ (b) Outside

9. Centre $(-2,3)$, radius $2\sqrt{2}$; Outside

10.
- The perpendicular bisectors of AB and BC are respectively $y = -x + 8$ and $5x - 9y - 5 = 0$.
- $(x, y) = (\frac{11}{2}, \frac{5}{2})$
- $\frac{1}{2}\sqrt{106}$
- $\left(x - \frac{11}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{106}{4}$, or equivalently $(2x-11)^2 + (2y-5)^2 = 106$

11. Solve $(1-a)^2 + (5-b)^2 = r^2$, $(3-a)^2 + (7-b)^2 = r^2$ and $(8-a)^2 + (-2-b)^2 = r^2$ to give $a = \frac{11}{2}, b = \frac{5}{2}$ and $r = \frac{1}{2}\sqrt{106}$.

12. (i) Centre $(3,-2)$, radius 4 ; Outside (ii) Centre $(-3, \frac{7}{2})$, radius $\frac{1}{2}\sqrt{133}$; Inside

13. (a) $(x, y) = \left(\frac{1}{5}(3 + \sqrt{154}), -\frac{1}{5}(31 + 2\sqrt{154})\right)$ and $\left(\frac{1}{5}(3 - \sqrt{154}), -\frac{1}{5}(31 - 2\sqrt{154})\right)$

(b) $(x, y) = \left(-\frac{1}{25}(6\sqrt{14} + 23), -\frac{1}{50}(9\sqrt{14} + 22)\right)$ or $\left(\frac{1}{25}(6\sqrt{14} - 23), \frac{1}{50}(9\sqrt{14} - 22)\right)$

(c) $(x, y) = \left(\dfrac{3(\sqrt{241} + 71)}{100}, \dfrac{21\sqrt{241} + 391}{100}\right)$ or $\left(\dfrac{3(71 - \sqrt{241})}{100}, \dfrac{391 - 21\sqrt{241}}{100}\right)$

14. $(x, y) = \left(-\frac{1}{9}(\sqrt{70} + 5), -\frac{1}{9}(2\sqrt{70} + 1)\right)$ or $\left(\frac{1}{9}(\sqrt{70} - 5), \frac{1}{9}(2\sqrt{70} - 1)\right)$

15. $y = x - 7$

16.

Angle (Degrees)	Angle (Radians)	Ratio					
		sin	***cos***	***tan***	***cosec***	***sec***	***cot***
0	0	0	1	0	∞	1	∞
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
90	$\frac{\pi}{2}$	1	0	∞	1	∞	0
180	π	0	-1	0	∞	-1	∞

17. (a) $AC \approx 10.1,\ BC \approx 15.7,\ \hat{C} = 50^0$ (b) $AB \approx 14.3,\ BC \approx 18.7,\ \hat{C} = 50^0$
(c) $AB = 4\sqrt{5},\ \hat{B} \approx 41.8^0,\ \hat{C} \approx 48.2^0$

18. (a) $x = t,\ y = 4t - 13$ (b) $x = t,\ 5y = 8t + 14$

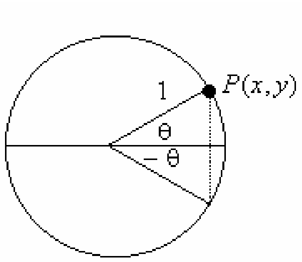
19. (i) $y = \frac{2}{21}(9x - 4)$ (ii) 41^o

20. (a) $y = \frac{1}{8}(9x^2 + 4x + 23)$ (b) $y = -\frac{1}{5}(1 + 2x)$

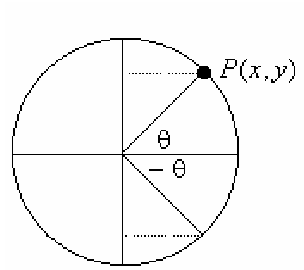
21. Closest approach is $13\sqrt{2}$ km, after 3 hours.

22. $x = \cos(6\pi t)$, $y = \sin(6\pi t)$

23.



$\cos\theta$ is the projection of $P(x, y)$ onto the x-axis.
Hence $\cos(-\theta) = \cos(\theta)$



$\sin\theta$ is the projection of $P(x, y)$ onto the y-axis.
Hence $\sin(-\theta) = -\sin(\theta)$

Hence $\tan(-\theta) = \dfrac{\sin(-\theta)}{\cos(-\theta)} = \dfrac{-\sin\theta}{\cos\theta} = -\tan\theta$

24. (a) $x = 4 + 5\cos t,\ y = 3 + 5\sin t,\ \arctan(\frac{4}{3}) \le t \le \pi + \arctan(\frac{4}{3})$

26. The curve intersects the x-axis at $(1 \pm \sqrt{3}, 0)$, the vertex is at $(1,3)$ and the equation of the axis of symmetry is $x = 2$.